

Performance study of a Missile Autopilot for various state feedback models

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Abstract— A missile autopilot with incomplete state feedback system is considered. A cost effective model has been proposed which assures desired transient performance. A complete performance of the system like time response characteristics has been studied here considering various flight parameters such as elevator deflection, missile body rate in pitch and missile flight path rate. The design ensures to achieve a certain desired stability margin. Thus a choice of a missile autopilot configuration can be done. The performance indices of the system have been executed in the MATLAB/SIMULINK environment to get the different characteristics.

Index Terms— Autopilot, elevator deflection, incomplete system, pitch plane, state feedback, MATLAB, simulation

1 INTRODUCTION

A Guided missile is one which receives steering commands from the guided system to improve its accuracy. Guidance system actually gives command to the autopilot to activate the controls to achieve the correction necessary. Autopilot is an automatic control mechanism for keeping the spacecraft in desired flight path. An autopilot in a missile is a close loop system and it is a minor loop inside the main guidance loop. If the missile carries accelerometer and rate gyros to provide additional feedback into the missile servos to modify the missile motion then the missile control system is usually called an autopilot. When the autopilot controls the motion in the pitch or yaw plane, they are called Lateral Autopilot. For a symmetrical cruciform missile, pitch and yaw autopilots are identical. The guidance system detects whether the missile's position is too high or too low, or too much right or left. It measures the deviation or errors and sends signals to the control system to minimize the acceleration (latex) according to the demand from the guidance computer. A systematic design methodology for the linear design of a lateral two loop autopilot for a class of guided missile which controls the lateral acceleration of the missile body using measurement from an accelerometer for output feedback and from a rate gyro to provide additional damping has been presented in [1]. The pole assignment techniques employed in the findings of researchers [4-5] utilised full state feedback.

A pole placement method for designing a linear missile autopilot for tail controlled missile has been presented in [2], which can provide fast and stable acceleration in pitch plane with good tracking quality. An incomplete state feedback controller has been designed and a numerical example illustrates the effectiveness of the developed methodology.

The main objective of the present work is to compare the parameters of the incomplete autopilot system [3, 6] to meet the performance stability of flight condition near the equilibrium point also keeping in mind the cost of the system. Thus it is necessary to design controllers that has stability robustness for optimum performance with minimum cost for different needs/modes of operation.

2 TWO LOOP MISSILE AUTOPILOT CONFIGURATION IN PITCH PLANE

The autopilot uses one accelerometer and one rate gyro [1]. The flight path rate demand autopilot is shown in Fig. 1. The transfer function which forms the basis for this two loop autopilot configuration are $G3(s)$, $G1(s)$ and $G2(s)$. Thus, the autopilot configuration in Fig. 1 is a modified form and is of flight path rate demand type instead of the conventional configuration with a lateral acceleration demand.

The missile state model is based upon the two loop configuration. Where, $G1(s)$ and $G2(s)$ are aerodynamic transfer function and $G3(s)$ represents the second order actuator. K_p is the control gain, γ is the input to the autopilot and $\dot{\gamma}$ is the output of the autopilot.

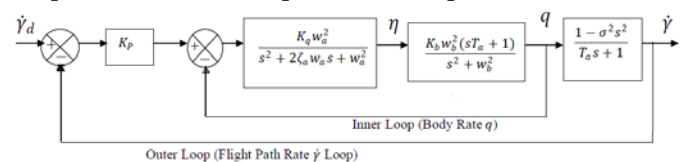


Figure 1: Block Diagram of two loop missile autopilot in pitch plane

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Autopilot System Design Parameters [3]:

The autopilot system design parameters for the missile have been given below.

Parameters	Values
T_a	2.85 sec
ω_b	5.6rad/sec
σ^2	0.00142 sec ²
ω_a	180rad/sec
ξ_a	0.6
K_b	-0.1437 sec ⁻¹
K_q	-1.72

Table 1: Two loop autopilot system design parameters.

Results that has been obtained by MATLAB simulations using parameter values and transfer functions are

$$G_1(s) = \frac{K_b \omega_b^2 (1+st_a)}{s^2 + \omega_b^2} = \frac{-(12.8433s+4.506432)}{(s^2+31.36)}$$

$$G_2(s) = \frac{(1-\sigma^2s)}{(1+st_a)} = \frac{(1-0.00142s^2)}{(1+2.85s)}$$

$$G_3(s) = \frac{K_q \omega_a^2}{s^2 + 2\xi_a \omega_a s + \omega_a^2} = \frac{-55728}{s^2 + 216s + 32400}$$

3 STATE FEEDBACK AUTOPILOT DESIGN

Since the state variable η (fin rate) is assumed to be not available, an incomplete state feedback controller [4] has been designed in Fig. 2 which uses a linear combination of the output and the two available state variables only to meet the desired autopilot specification in terms of gain margin and phase margin and a unity steady state gain. Therefore, three control gains have been used to move the closed loop poles to any desired locations.

Poles assignment: Denoting the chosen closed-loop pole locations as

$$s_{1,2} = -a \pm jb \text{ (dominant poles),}$$

$$s_{3,4} = -c \pm jd \text{ (faster poles),}$$

The desired characteristic equation is

$$s^4 + d_3 s^3 + d_2 s^2 + d_1 s + d_0 = 0 \quad \text{[from the equation (1)]}$$

Where,

$$d_3 = (2a + 2c) = 216 \quad (2)$$

$$d_2 = (a^2 + b^2 + c^2 + d^2 + 4ac) = 1.5360 \times 10^4 \quad (3)$$

$$d_1 = (2a^2c + 2b^2c + 2ac^2 + 2ad^2) = 4.7166 \times 10^5 \quad (4)$$

$$d_0 = (a^2 + b^2)(c^2 + d^2) = 5.5786 \times 10^6 \quad (5)$$

Where,

$$a = \frac{\xi_a \omega_a}{6} = 18$$

$$b = \frac{-\pi \xi_a \omega_a}{6 \ln(M_p)} = 18.8759$$

$$c = \frac{5\xi_a \omega_a}{6} = 90$$

$$d = 10$$

The control gain matrix $K_T = [k_1 \ k_2 \ k_3 \ k_4]$ may be obtain from Ackermann's formula once the desired closed loop poles are specified. The elements of the control gain matrix in terms of aerodynamic parameters and the actuator parameters are given as

$$k_1 = \frac{(d_0 - \omega_a^2 b^2)T_a \sigma^2 - (d_1 - 2\xi_a \omega_a \omega_b^2)\sigma^2 + (d_2 - \omega_a^2 - \omega_b^2)T_a + (d_3 - 2\xi_a \omega_a)\omega_b^2 \sigma^2}{k_s k_q \omega_a^2 T_a (1 + \sigma^2 \omega_b^2)} \quad (6)$$

$$k_2 = \frac{d_3 - 2\xi_a \omega_a}{k_s k_q \omega_a^2} \quad (7)$$

$$k_3 = \frac{(d_1 - 2\xi_a \omega_a \omega_b^2) - (d_3 - 2\xi_a \omega_a)\omega_b^2}{k_s k_q k_b \omega_a^2 \omega_b^2 T_a} \quad (8)$$

$$k_4 = \frac{(d_0 - \omega_a^2 b^2)T_a - (d_1 - 2\xi_a \omega_a \omega_b^2) - (d_2 - \omega_a^2 - \omega_b^2)T_a \omega_b^2 + (d_3 - 2\xi_a \omega_a)\omega_b^2}{k_s k_q k_b \omega_a^2 \omega_b^2 T_a (1 + \sigma^2 \omega_b^2)} \quad (9)$$

Shifting of the closed-loop poles can be effected without implementing any feedback from the state η (fin rate) by choosing $d_3 = 2\xi_a \omega_a$ in equation (7) such that the control gain $k_2 = 0$ for all operating conditions.

Substituting d_3 in equations (2), we can get

$$a + c = \xi_a \omega_a \quad (10)$$

For using the incomplete sate feedback configuration, equation (10) sets the condition on the magnitudes of the real parts of the closed loop pole pairs for $k_2 = 0$.

For $d_3 = 2\xi_a \omega_a$, equations (6-9) are modified to

$$k_1 = \frac{(d_0 - \omega_a^2 b^2)T_a \sigma^2 - (d_1 - 2\xi_a \omega_a \omega_b^2)\sigma^2 + (d_2 - \omega_a^2 - \omega_b^2)T_a}{k_s k_q \omega_a^2 T_a (1 + \sigma^2 \omega_b^2)} \quad (11)$$

$$k_2 = 0 \quad (12)$$

$$k_3 = \frac{(d_1 - 2\xi_a \omega_a \omega_b^2)}{k_s k_q k_b \omega_a^2 \omega_b^2 T_a} \quad (13)$$

$$k_4 = \frac{(d_0 - \omega_a^2 b^2) T_a - (d_1 - 2\xi_a \omega_a \omega_b^2) - (d_2 - \omega_a^2 - \omega_b^2) T_a \omega_b^2}{k_s k_q k_b \omega_a^2 \omega_b^2 T_a (1 + \sigma^2 \omega_b^2)} \quad (14)$$

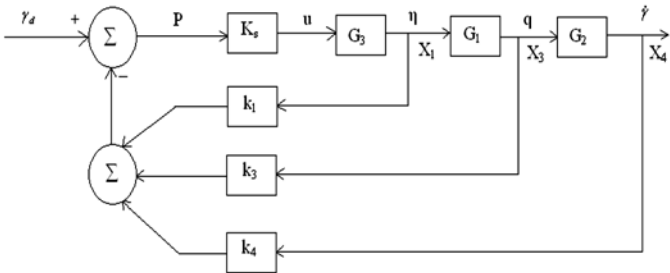


Figure 2: Incomplete state feedback configuration of missile autopilot in pitch plane

4 DESIGN ANALYSIS

Time responses of incomplete state feedback system has been studied and also simultaneously removing the feedback loops of η (elevator deflection), q (missile body rate in pitch) and $\dot{\gamma}$ (missile flight path rate), a comparative study has been done for the time responses of above mentioned cases.

Method Analysis: To evaluate the performance of the incomplete state feedback controller, the time domain analysis have been done by MATLAB programming. The best result for each case has been obtained by altering main loop gain, K_s , by trial and error method.

Firstly, removing the feedback loop of η (elevator deflection)

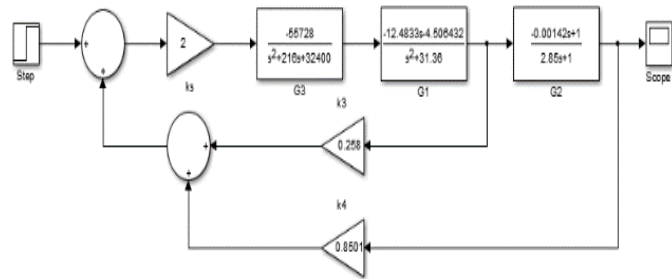


Figure 3: Simulink model of autopilot in incomplete state feedback while removing the feedback loop of η

Secondly, removing the feedback loop of q (missile body rate in pitch plane)

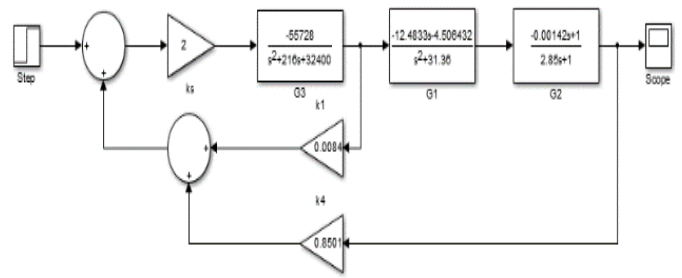
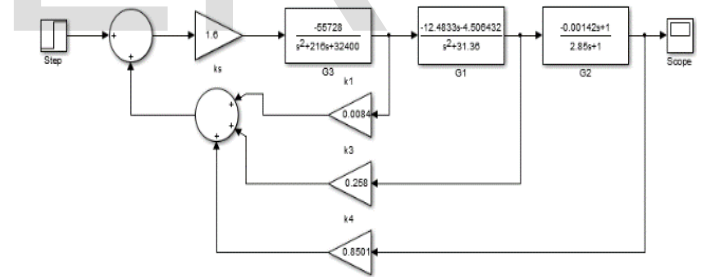


Figure 4: Simulink model of autopilot in incomplete state feedback while removing the feedback loop of q

Finally, removing the feedback loop of $\dot{\gamma}$ (missile flight path rate)

Figure 5: Simulink model of autopilot in incomplete state



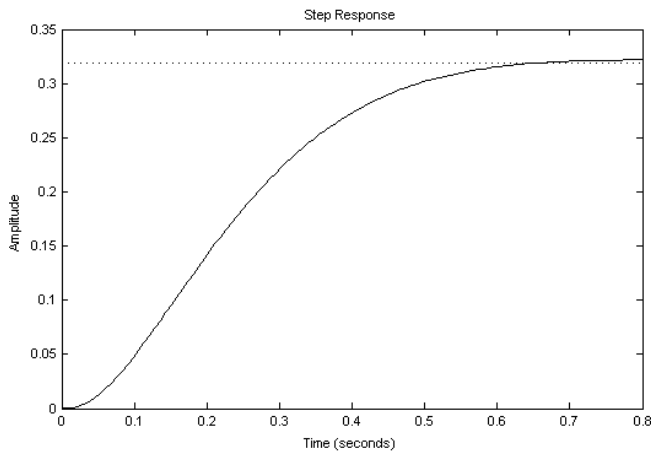
feedback while removing the feedback loops of $\dot{\gamma}$

Now, the time response of the entire incomplete state feedback system without removing any loops has been shown below.

Figure 6: Simulink model of autopilot in incomplete state feedback

5 PERFORMANCE ANALYSIS

The performance study for the time response of various cases of incomplete state feedback system mentioned in the



previous section has been assessed below. Firstly, the step response obtained by removing the feedback loop of η (elevator deflection) is:

Figure 7: Step response of autopilot in incomplete state feedback while removing the feedback loops of η

The time response obtained for the least settling time when $K_S = 1.84$ is,

Settling Time (ts) = 0.517 seconds
 Rise Time (tr) = 0.337 seconds

Secondly, the step response obtained by removing the feedback loop of q (missile body rate in pitch plane) is:

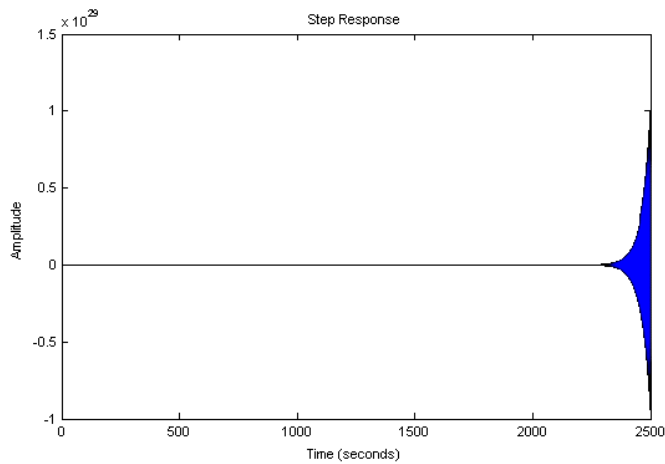


Figure 8: Step response of autopilot in incomplete state feedback while removing the feedback loops of q

In this case there is no suitable time responses obtained by altering the main loop gain K_S at any value.

Finally, the step response obtained by removing the feedback loop of $\dot{\gamma}$ (missile flight path rate) is:

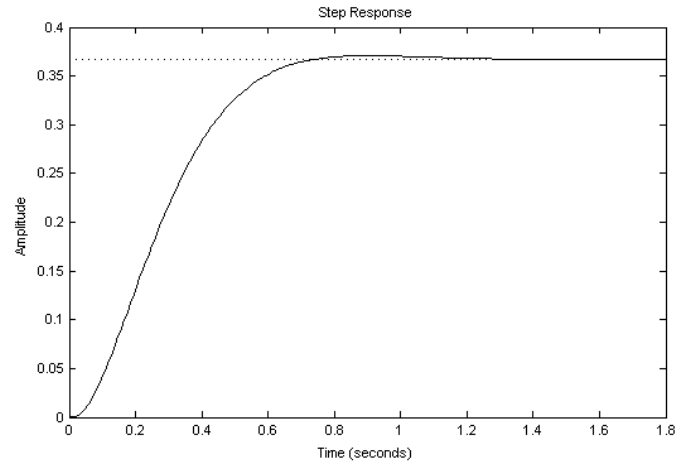


Figure 9: Step response of autopilot in incomplete state feedback while removing the feedback loops of $\dot{\gamma}$

The time response obtained for the least settling time when $K_S = 1.6$ is,

Settling Time (ts) = 0.649 seconds
 Rise Time (tr) = 0.417 seconds

Also, the step response obtained from incomplete state feedback system without removing any loops is:

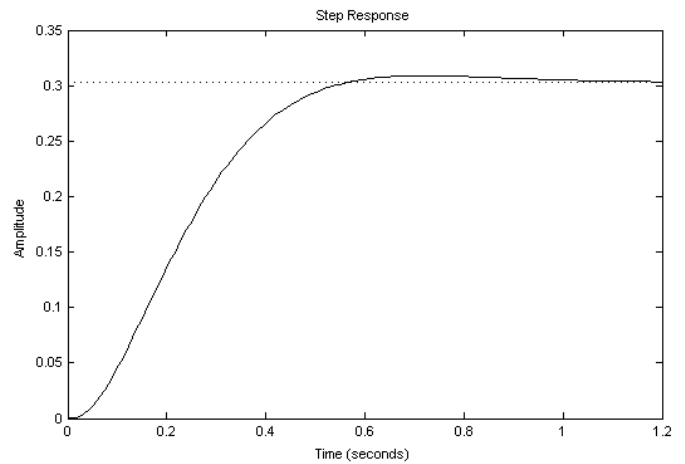


Figure 10: Step response of autopilot in incomplete state feedback

The time response obtained for the least settling time when $K_S = 1.79$ is,

Settling Time (ts) = 0.516 seconds
 Rise Time (tr) = 0.337 seconds

Table: Tabulated data for all the above mentioned performance analysis is given below.

5.1 Results of performance analysis of three different cases for the incomplete state feedback system

Incomplete State Feedback System	Main Loop Gain, K_s	Settling Time, t_s (seconds)	Rise Time, t_r (seconds)
Removing k_1	0.1	13.4	0.191
	0.5	2.75	0.208
	1	1.3	0.241
	1.83	0.735	0.335
	1.84	0.517	0.337
	1.85	0.52	0.339
	2	0.574	0.363
Removing k_3	0.1	N/A	N/A
	1	N/A	N/A
	10	N/A	N/A
	20	N/A	N/A
Removing k_4	0.1	13.5	0.193
	1	1.36	0.281
	1.3	1.01	0.337
	1.5	0.925	0.387
	1.6	0.649	0.417
	1.7	0.724	0.449
	2	0.993	0.561

Table 2: Time response data for Incomplete State Feedback System eliminating each loops

5.2 RESULTS OF THE PERFORMANCE ANALYSIS FOR INCOMPLETE STATE FEEDBACK SYSTEM WITHOUT ELIMINATING ANY LOOPS

Incomplete State Feedback System	Main Loop Gain, K_s	Settling Time, t_s (seconds)	Rise Time, t_r (seconds)
Keeping all the loops	0.1	13.4	0.191
	0.5	2.73	0.208
	1	1.3	0.242

	1.5	0.882	0.295
	1.7	0.826	0.323
	1.78	0.74	0.335
	1.79	0.516	0.337
	1.8	0.519	0.338
	1.9	0.557	0.355
	2	0.6	0.372

Table 3: Time response data for Incomplete State Feedback System keeping all the loops

6 CONCLUSION

The performance of incomplete state feedback controller has been studied for different feedback paths consisting of different sensors and also keeping all the feedback paths i.e., keeping all the sensors. A total of four different separate cases has been studied and their time responses are analysed. It is found that by keeping q (missile body rate in pitch) sensor and $\dot{\gamma}$ (missile flight path) sensors of the system, and compromising η (elevator deflection), an improved settling time of 0.517 seconds with a rise time of 0.337 seconds is obtained among three different cases that has been analysed by removing different loops of incomplete state feedback system one at a time.

But while analysing incomplete state feedback system by not removing any of the loops, then we observe a little bit of improved settling time of 0.516 seconds with a rise time of 0.337 seconds but it is quite negligible.

So it is observed that an incomplete state feedback system with no elevator deflection reduces the cost of the entire system with almost same settling time and rise time compared to the system which consists of sensor of elevator deflection. A cost effective design with the absence of elevator deflection has been achieved in this paper for a specific operational need.

7 NOTATIONS

- K_p : lateral autopilot control gain outer loop
- K_q : fin servo gain, s-1
- K_b : airframe aerodynamic gain, s-1
- q : missile body rate in pitch, rad/sec
- T_a : incidence lag of airframe, s
- η : elevator deflection, rad
- $\dot{\gamma}$: missile flight path rate, rad/sec

- $\dot{\gamma}_d$: missile flight path rate demand, rad/sec
- ω_a : natural frequency of oscillation of actuator, rad/sec
- ω_b : weather cock frequency, rad/sec
- ξ_a : damping ratio of actuator
- σ : a quantity whose inverse determines the location of non-minimum phase zeros in s-plane

8 REFERENCES

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